Fractions at Stage 7

Finding Fractions
1. _____ equals one half
2. _____ is less than one
3. _____ is greater than 1
4. _____ cannot be simplified
5. _____ is less than one half
6. _____ is greater than one half
7. _____ equals one
8. _____ equals one third
9. _____ is equal to two
10. _____ equals a whole number

Finding Fractions

Throw 2 dice and make a fraction,

e.g. 4 and 5 could be 4 fifths of 5 quarters.

Try and make a true statement each time the dice is thrown.

Throw dice 10 times, Miss a go if you cannot place a fraction.

Making sense with equivalent fractions

Objectives:

- Explore how prevalent fractional equivalence is in Stages 7 and 8.

- Understand equivalent fractions in order to
  add and subtract with fractions
  and multiply and divide with fractions

- Become familiar with some resources, activities to help with teaching Stage 7 & 8 Fractions.

- Make sense of, and apply contexts to, e.g.
  $^{2}/_{3} \times {}^{4}/_{5}$ and $2\frac{1}{2} \div \frac{1}{2}$

Stage 7 (AM) Key Ideas (level 4)

Fractions and Decimals

- Rename improper fractions as mixed numbers, e.g. $^{17}/_{3} = 5^{2}/_{3}$
- Find equivalent fractions using multiplicative thinking, and order fractions using equivalence and benchmarks. e.g. $^{2}/_{5} < {}^{11}/_{16}$
- Convert common fractions, to decimals and percentages and vice versa.
- Add and subtract related fractions, e.g. $^{2}/_{4} + {}^{5}/_{8}$
- Add and subtract decimals, e.g. 3.6 + 2.89
- Find fractions of whole numbers using multiplication and division e.g. $^{2}/_{3}$ of 36 and $^{2}/_{3}$ of ? = 24
- Multiply fractions by other factions e.g. $^{2}/_{3} \times ¼$
- Solve measurement problems with <u>related</u> fractions, e.g. 1½ ÷ $^{1}/_{6} = {}^{9}/_{6} \div {}^{1}/_{6}$ =9
- Solve division problems expressing remainders as fractions or decimals e.g. 8 ÷ 3 = $2^{2}/_{3}$ or 2.66

Percentages

- Estimate and solve percentage type problems such as 'What % is 35 out of 60?', and 'What is 46% of 90?' using benchmark amounts like 10% and 5%

Ratios and Rates

- Find equivalent ratios using multiplication and express them as equivalent fractions, e.g. 16:8 as 8:4 as 4:2 as 2:1 = $^{2}/_{3}$
- Begin to compare ratios by finding equivalent fractions, building equivalent ratios or mapping onto 1).
- Solve simple rate problems using multiplication, e.g. Picking 7 boxes of apples

Stage 7 (AM) Key Ideas (level 4)

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Stage 8 (AP) Key Ideas (level 5)

Fractions and Decimals

- Add and subtract fractions and mixed numbers with uncommon denominators, $^{2}/_{3} + {}^{14}/_{8}$
- Multiply fractions, and divide whole numbers by fractions, recognising that division can result in a larger answer, e.g. $4 \div {}^{2}/_{3} = {}^{12}/_{3} \div {}^{2}/_{3} = 6$
- Solve measurement problems with fractions like $¾ \div {}^{2}/_{3}$ by using equivalence and reunitising the whole
- Multiply and divide decimals using place value estimation and conversion to known fractions, e.g. $0.4 \times 2.8 = 1.12$ ($0.4 < ½$) , $8.1 \div 0.3 = 27$ ($81 \div 3$ in tenths)
- Find fractions between two given fractions using equivalence, conversion to decimals or percentages

Percentages

- Solve percentage change problems, e.g. The house price rises from $240,000 to $270,000. What percentage increase is this?
- Estimate and find percentages of whole number and decimal amounts and calculate percentages from given amounts e.g. Liam gets 35 out of 56 shots in. What percentage is that?

Ratios

- Combine and partition ratios, and express the resulting ratio using fractions and percentages, e.g. Tina has twice as many marbles as Ben. She has a ratio of 2 red to 5 blue. Ben's ratio is 3:4. If they combine their collections what will the ratio be? i.e. 2:5 + 2:5 + 3:4 = 7:14 = 1:2 ,
- Find equivalent ratios by identifying common whole number factors and express them as fractions and percentages, e.g. 16:48 is equivalent to 2:6 or 1:3 or ¼ or 25%

Rates:

- Solve rate problems using common whole number factors and convertion to unit rates, e.g. 490 km in 14 hours is an average speed of 35 k/h (dividing by 7 then 2).
- Solve inverse rate problems, e.g. 4 people can paint a house in 9 days. How long will 3 people take to do it?

Stage 8 (AP) Key Ideas (level 5)

Fractions and Decimals

- Add and subtract fractions and mixed numbers with uncommon denominators, $^{2}/_{3} + {}^{14}/_{8}$
- Multiply fractions, and divide whole numbers by fractions, recognising that division can result in a larger answer, e.g. $4 \div {}^{2}/_{3} = {}^{12}/_{3} \div {}^{2}/_{3} = 6$
- Solve measurement problems with fractions like ¾ ÷ $^{2}/_{3}$ by using equivalence and reunitising the whole
- Multiply and divide decimals using place value estimation and conversion to known fractions, e.g. 0.4 × 2.8 = 1.12 (0.4< ½) , 8.1 ÷ 0.3 = 27 (81÷ 3 in tenths)
- Find fractions between two given fractions using equivalence, conversion to decimals or percentages

Percentages

- Solve percentage change problems, e.g. The house price rises from $240,000 to $270,000. What percentage increase is this?
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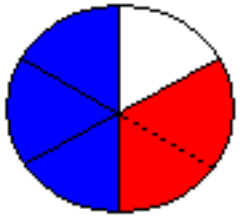
Equivalent Fractions

How could you communicate this idea of equivalence to students?

Fraction Circles

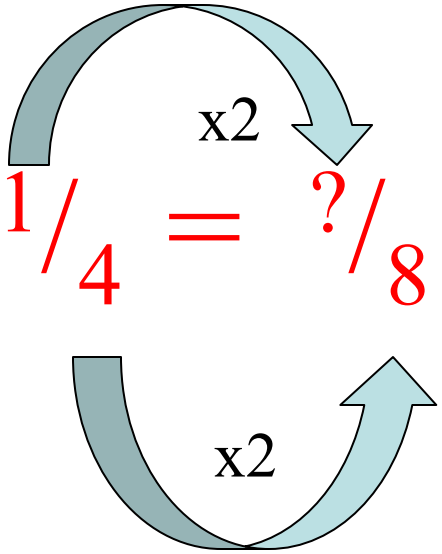


Paper Folding

$\frac{1}{4}$	

$\frac{1}{8}$	$\frac{1}{8}$		

Multiplicative thinking



Fraction Tiles / Strips

$\frac{1}{3}$		$\frac{1}{3}$		$\frac{1}{3}$	
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{12}$ $\frac{1}{12}$	$\frac{1}{12}$ $\frac{1}{12}$	$\frac{1}{12}$ $\frac{1}{12}$	$\frac{1}{12}$ $\frac{1}{12}$	$\frac{1}{12}$ $\frac{1}{12}$	$\frac{1}{12}$ $\frac{1}{12}$

Using Decipipes

1. establish the whole, half, quarter rods then tenths
2. 1 half = ? tenths
3. 1 quarter = ? tenths +
4. 1 eighth = ? tenths? +

Once you understand equivalence you can……

1. Compare and order fractions
2. Add and Subtract fractions
3. Understand decimals, as decimals are special cases of equivalent fractions where the denominator is always a power of ten.

Circle the bigger fraction of each pair.

What did you do to order them?

A	B	C	D
½ or ¼	$^{6}/_{4}$ or $^{3}/_{5}$	$^{7}/_{16}$ or $^{3}/_{8}$	$^{7}/_{10}$ or $^{6}/_{8}$
$^{1}/_{5}$ or $^{1}/_{9}$	$^{7}/_{8}$ or $^{9}/_{7}$	$^{2}/_{3}$ or $^{5}/_{9}$	$^{7}/_{8}$ or $^{6}/_{9}$
$^{5}/_{9}$ or $^{2}/_{9}$	$^{7}/_{3}$ or $^{4}/_{6}$	$^{5}/_{4}$ or $^{3}/_{2}$	$^{5}/_{7}$ or $^{7}/_{9}$
unit fractions	More or less than 1	related fractions	unrelated fractions

Key Idea
Ordering using equivalence and benchmarks

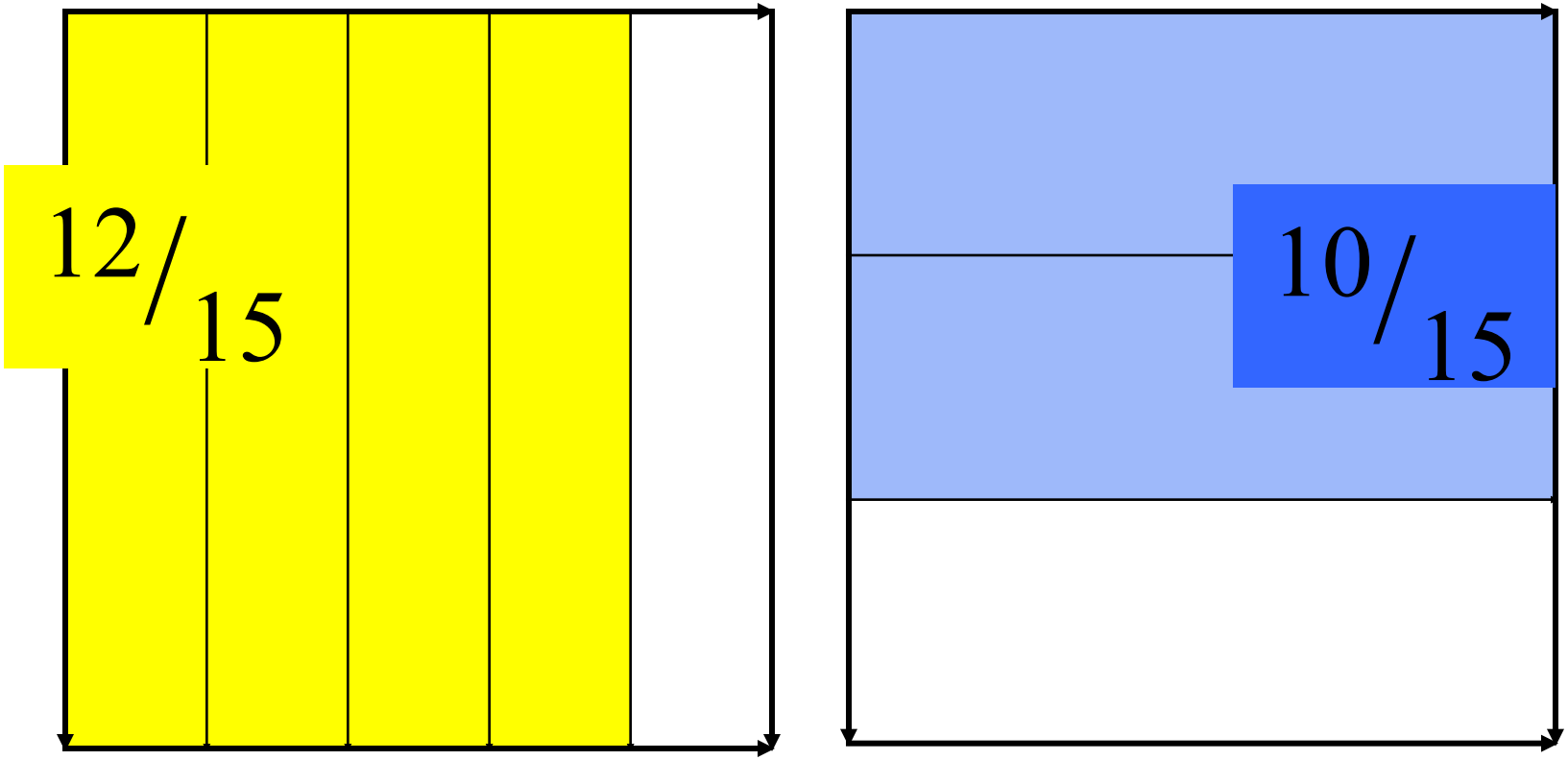
Example of Stage 8 fraction knowledge

$^{2}/_{3}$ $^{3}/_{4}$ $^{2}/_{5}$ $^{5}/_{8}$ $^{3}/_{8}$

Which is bigger?

(Order/compare fractions: Stage 7)

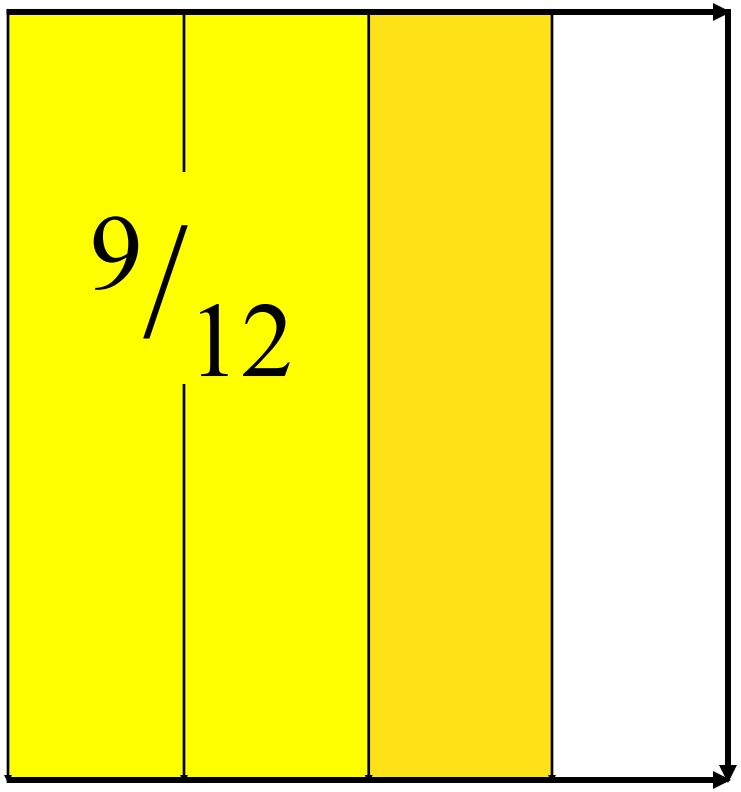
4/5 or 2/3



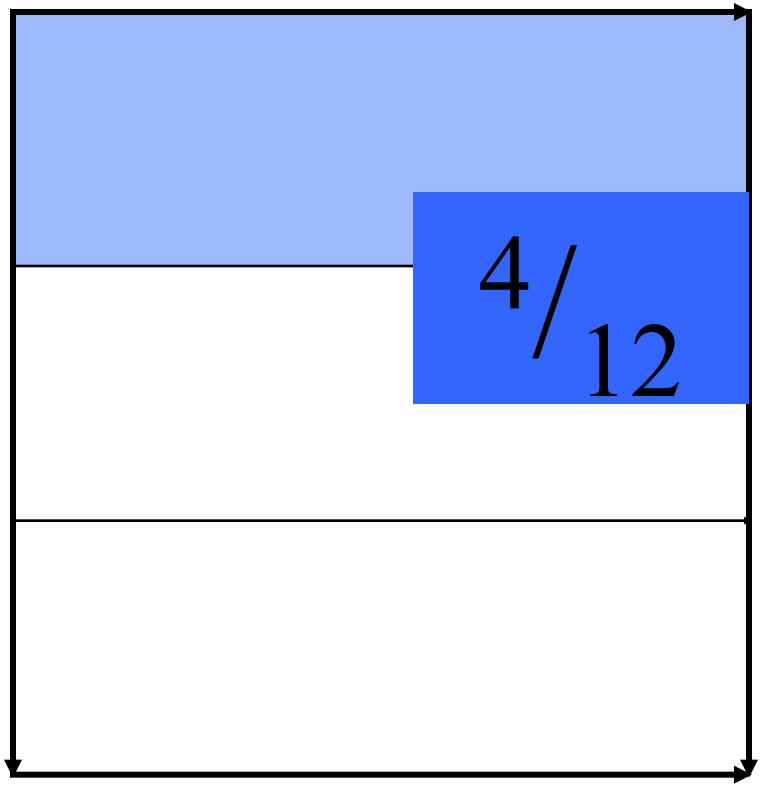
Find fractions between two fractions, using equivalence:

What fractions come between these two?

3/4



1/3



Stage 8 Feeding Pets (Book 7, p67)

Add and Subtract <u>related</u> fractions

(Stage 7)

e. g ¼ + $^{5}/_{8}$

- halves, quarters, eighths
- halves, fifths, tenths
- halves, thirds, sixths

What could you use to help students understand this idea?

- Start with same denominator fractions.
- When using unlike denominators,

  key idea: rename one fraction so the denominators are the same

Comparing Apple with apples (Book 7, p65)

Use Fraction circles, strips, wall tiles

*Play create 3 (MM 7-9)

Add and Subtract fractions with uncommon denominators

(Stage 8)

e.g. $^{2}/_{3} + {}^{9}/_{4}$

- How??
- Find common denominators/ equivalent fractions **using number properties**

"Comparing Apples with Apples" Book 7, p65

Using fraction circles, strips, wall tiles

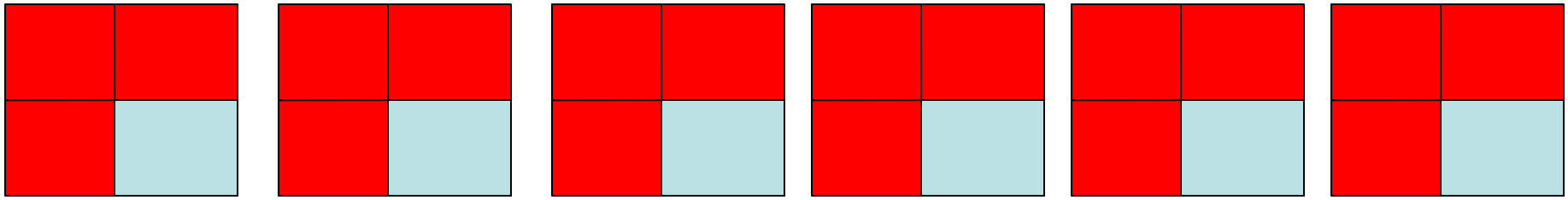
Play "Fractis"

Multiplying Fractions (Stage 7)

6 x ¾

Push towards multiplicative thinking, over additive

$= {}^{18}/_{4}$ $= 4\ {}^{2}/_{4}$ or $4\ {}^{1}/_{2}$



Using fraction circles, wall tiles

Whole Number Times Fractions (Book 8, p22)
Fractions Times Whole Numbers (Book 8, p23)

Multiplying Fractions (Stage 7)

½ x ¼

Using multiplicative thinking, not additive

Remember "x" means "lots of", "sets of", "quantities of"

Use paper folding, OHP overlays,

A Fraction Times a Fraction (Book 8, p24)

Pirate Problem

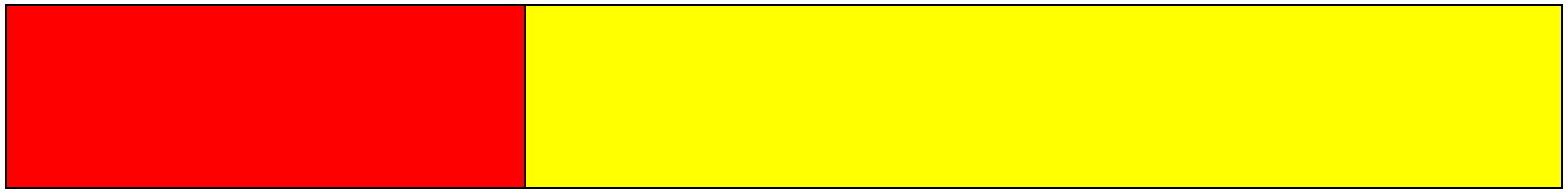
- Three pirates have some treasure to share. They decide to sleep and share it equally in the morning.
- One pirate got up at at 1.00am and took 1 third of the treasure.
- The second pirate woke at 3.00am and took 1 third of the treasure.
- The last pirate got up at 7.00am and took the rest of the treasure.

Do they each get an equal share of the treasure?

If not, how much do they each get?

Pirate Problem

- One pirate got up at at 1.00am and took 1 third of the treasure.
- The second pirate woke at 3.00am and took 1 third of the treasure.
- The last pirate got up at 7.00am and took the rest of the treasure.



1st pirate = 1 third

2nd pirate = $^{1}/_{3}$ x $^{2}/_{3}$
= 2 ninths

3rd pirate = the rest
= 1 - 5 ninths
= 4 ninths

Cake lovers!

Choose your share of cake!

Use the OHP transparencies to help.

Multiplying fractions

Jo ate $^1/_2$ of a box of chocolates she had for Mother's Day.
Her greedy husband ate 1/4 of what she left.
What fraction of the whole box is left?

$$\frac{1}{2} \times \frac{1}{4}$$

How might you help student understand this idea?

Multiplying fractions

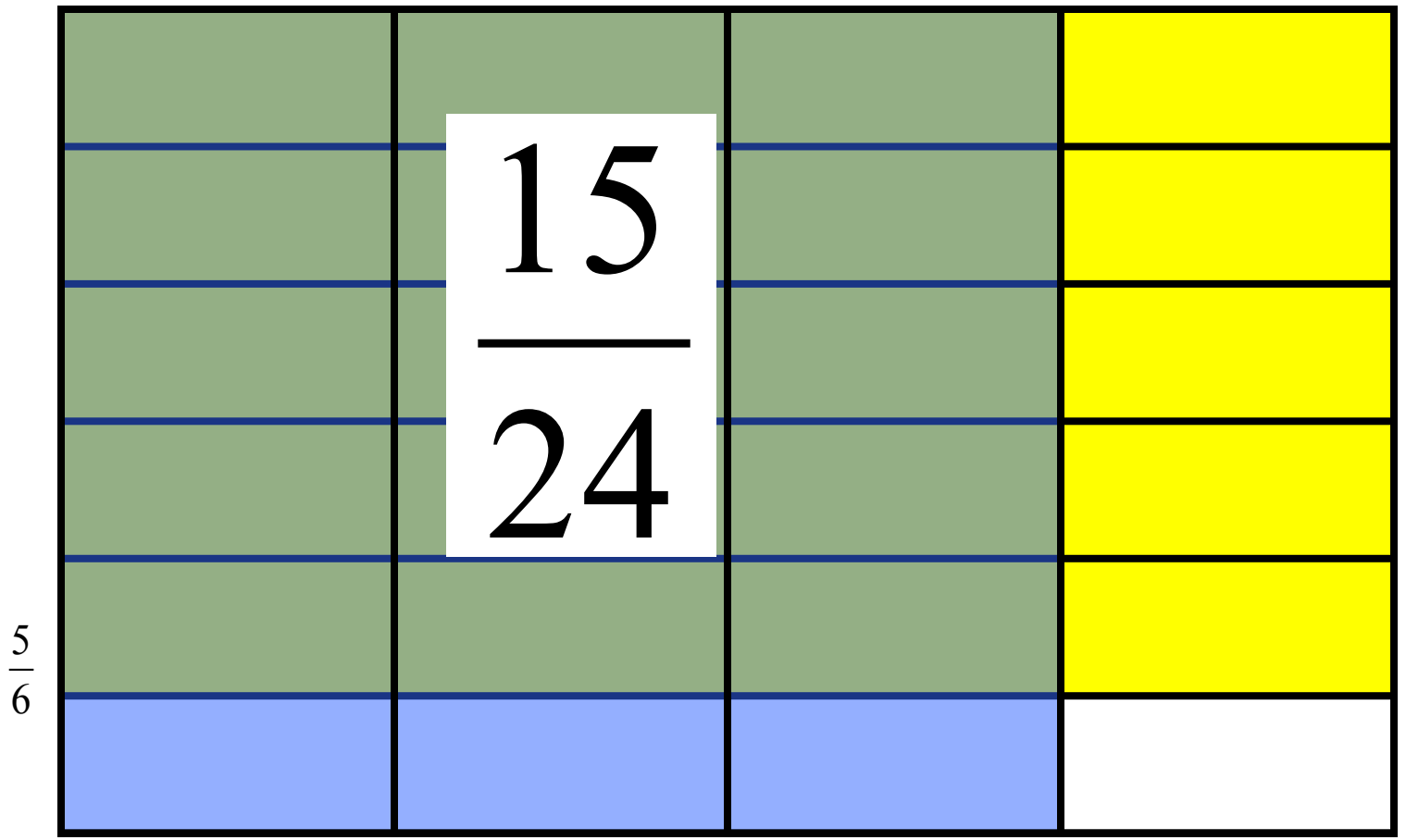
Jo ate $^1/_6$ of a box of chocolates she had for Mother's Day.
Her greedy husband ate ¾ of what she left.
What fraction of the whole box is left?

$$\frac{3}{4} \times \frac{5}{6}$$

How might you help student understand this idea?

Multiplying fractions

$$\frac{3}{4} \times \frac{5}{6}$$



Digital Learning Objects "Fractions of Fractions" tool

Multiplying fractions – your turn!

What is a word problem / context for:

$$\frac{3}{8} \times \frac{5}{6}$$

Draw a picture, or use the Fraction OHTs to represent the problem

Play "Fraction Multiplication grid" game

Dividing by fractions: Stage 7

Solve measurement problems with <u>related</u> fractions, (recognise than division can lead to a larger answer)

You observe the following equation in Bill's work:

$$2\frac{1}{2} \div \frac{1}{2} = 1\frac{1}{4}$$

Consider…..

- Is Bill correct?
- What is the possible reasoning behind his answer?
- What, if any, is the key understanding he needs to develop in order to solve this problem?

No he is not correct. The correct equation is

$$2\frac{1}{2} \div \frac{1}{2} = 5$$

Possible reasoning behind his answer:

$^1/_2$ of 2 $^1/_2$ is 1 $^1/_4$.
He is dividing by 2.
He is multiplying by 1/2.
He reasons that "division makes smaller" therefore the answer must be smaller than 2 1/2.

Key Idea:
To divide the number A by the number B is to find out **how many lots of B** are in A

$$8 \div 2 = 4$$

$$2\frac{1}{2} \div \frac{1}{2} = 5$$

For example:

There are 4 lots of 2 in 8

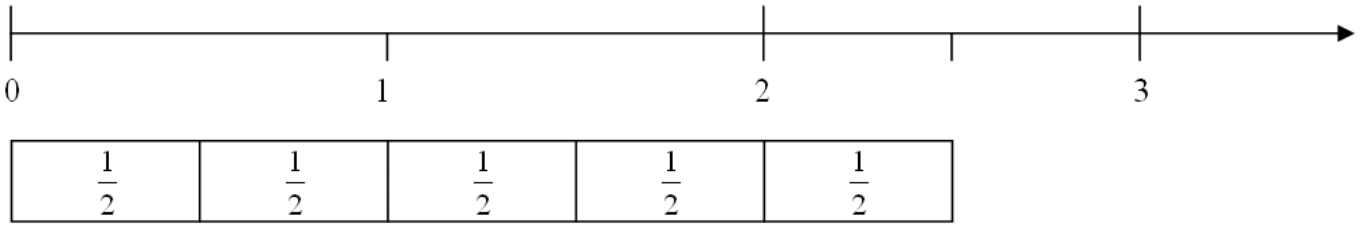
There are 5 lots of $^1/_2$ in 2 $^1/_2$

To communicate this idea to students you could...

- Use meaningful representations for the problem. For example:

  I am making hats. If each hat takes $^1/_2$ a metre of material, how many hats can I make from 2 $^1/_2$ metres?

- Use materials or diagrams to show there are 5 lots of $^1/_2$ in 2 $^1/_2$:



Key Idea:
Division is the opposite of multiplication.

The relationship between multiplication and division can be used to help simplify the solution to problems involving the division of fractions.

To communicate this idea to students you could…

Use contexts that make use of the inverse operation:

If $2\frac{1}{2} \div \frac{1}{2} = ?$ then $? \times \frac{1}{2} = 2\frac{1}{2}$

Your turn!

$4\frac{1}{2} \div 1\frac{1}{8}$ is

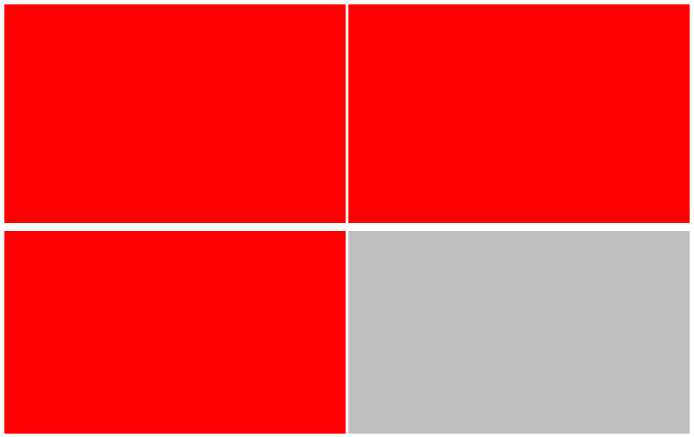
Remember the key idea is to think about how many lots of B are in A, or use the inverse operation…

Use materials or diagrams

Use contexts that make use of the inverse operation:

Example

Malcolm has ¾ of a cake left.
He gives his guests 1/8 of a cake each.
How many guests get a piece of cake?



$$\frac{3}{4} \div \frac{1}{8}$$

Example

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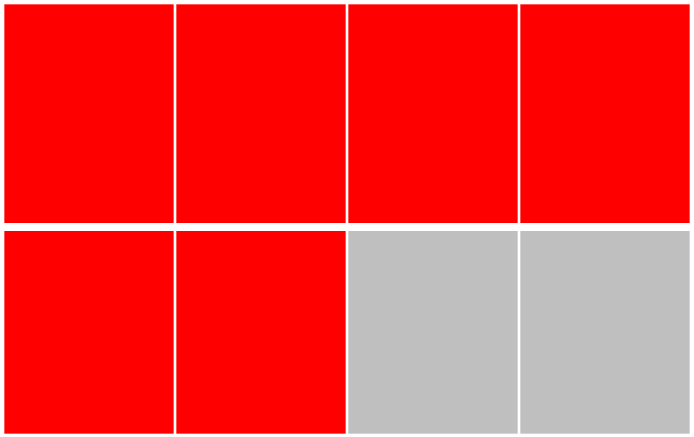


$\frac{3}{4} \div \frac{1}{8}$

Or, $\frac{6}{8} \div \frac{1}{8}$

How many one eighths in six eighths?...Answer 6

Stage 8 Advanced Proportional

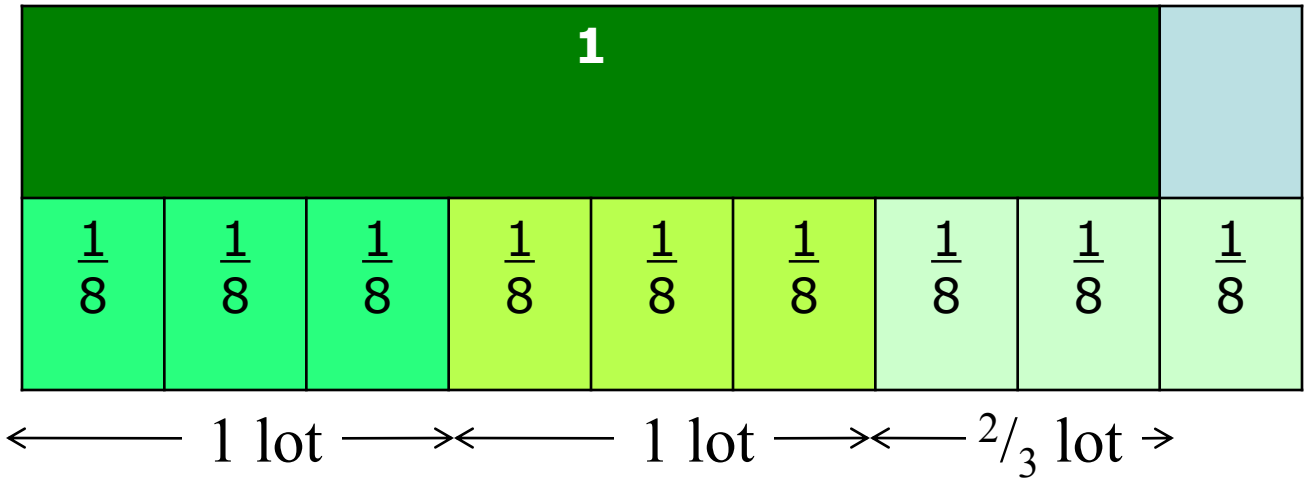
Solve measurement problems with fractions by using equivalence and reunitising the whole.

Ref: Book 7: p68, Brmmm! Brmmm!

Brmmm! Brmmm!

Book 7, p68

Trev has just filled his car.
He drives to and from work each day. Each trip takes three eighths of a tank. How many trips can he take before he runs out of petrol?



$1 \div {}^{3}/_{8}$

"How many three-eighths measure one whole?" $2\,{}^{2}/_{3}$

Stage 8 Advanced Proportional

Solve measurement problems with fractions by using equivalence and reunitising the whole.

Example

Why not 9/8 twelfths?

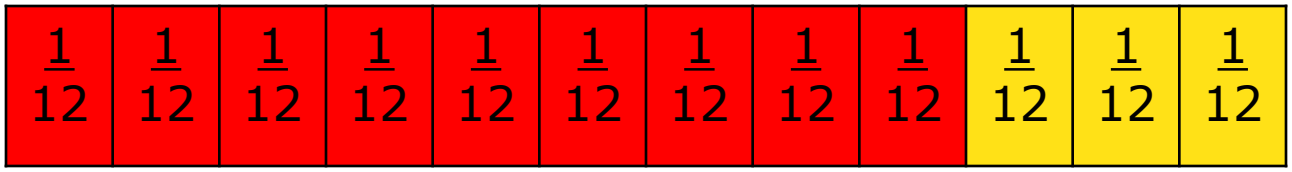
$$\frac{3}{4} \div \frac{2}{3} \rightarrow \frac{9}{12} \div \frac{8}{12} \rightarrow \frac{9}{8} \text{ (Or } 1\ {}^{1}/_{8}\text{)}$$

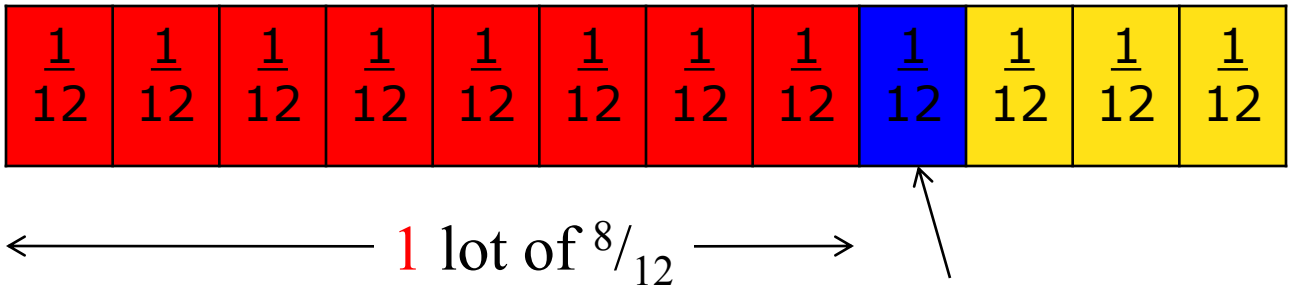
Ref: Book 8 : p21, Dividing Fractions
p22, Harder Division of Fractions

Why not $^{9}/_{8}$ twelfths?

$$\frac{3}{4} \div \frac{2}{3} \rightarrow \frac{9}{12} \div \frac{8}{12} \rightarrow \frac{9}{8} \text{ (Or } 1\ {}^{1}/_{8}\text{)}$$

How many times will $^{8}/_{12}$ go into $^{9}/_{12}$?

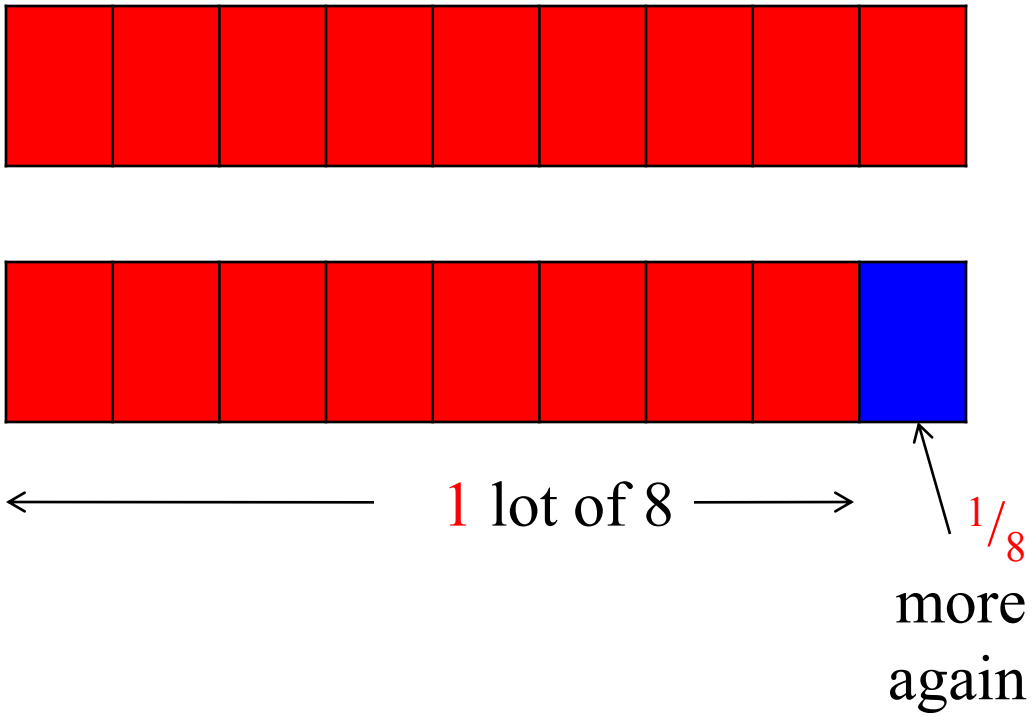




Why not $^{9}/_{8}$ twelfths?

$$\frac{9}{?} \div \frac{8}{?} \rightarrow \frac{9}{8}$$ (Or 1 $^{1}/_{8}$)

How many times will $^{8}/_{\text{somethings}}$ go into $^{9}/_{\text{somethings}}$?



Example

$$\frac{3}{4} \div \frac{2}{3} \rightarrow \frac{9}{12} \div \frac{8}{12} \rightarrow \frac{9}{8}$$

Your turn:

$$\frac{7}{8} \div \frac{1}{4}$$

$$\frac{3}{5} \div \frac{2}{8}$$

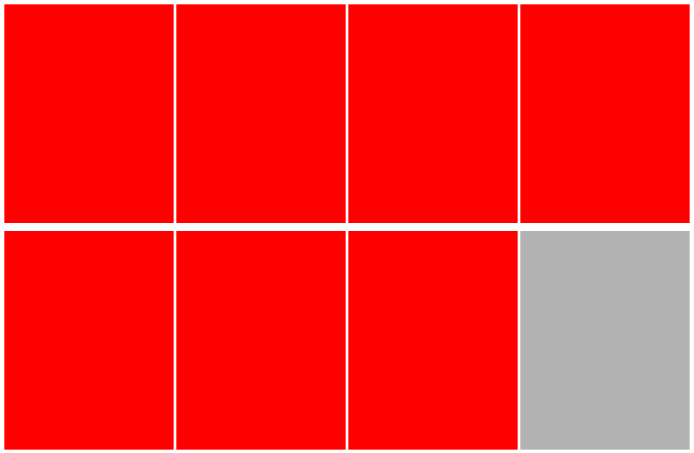
Make a word story/context for each problem.

Use pictures/diagrams to model

Harder Division of Fractions

Book 8, p22

Malcolm has $^{7}/_{8}$ of a cake left.
He cuts $^{2}/_{9}$ in size to put in packets for his guests.
How many packets of cake will he make?



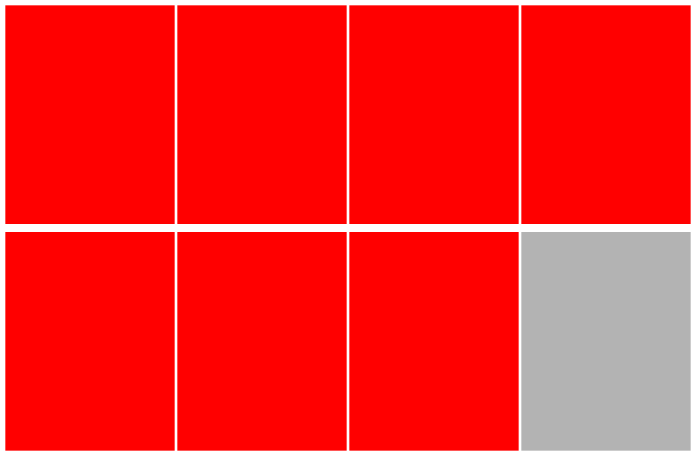
$$^{7}/_{8} \div {}^{2}/_{9}$$

Why is this hard to compare?

Harder Division of Fractions

Book 8, p22

Malcolm has $^{7}/_{8}$ of a cake left.
He cuts $^{2}/_{9}$ in size to put in packets for his guests.
How many packets of cake will he make?



Rewrite them as equivalent fractions

$^{63}/_{72} \div {}^{16}/_{72} \rightarrow 63 \div 16 \rightarrow {}^{63}/_{16}$ or $3\,{}^{15}/_{16}$

Chocoholic

You have three-quarters of a chocolate block left.

You usually eat one-third of a block each sitting for the good of your health.

How many sittings will the chocolate last?

Making sense with equivalent fractions

Objectives:

- Explore how prevalent fractional equivalence is in Stages 7 and 8.

- Understand equivalent fractions in order to
  add and subtract with fractions
  and multiply and divide with fractions

- Become familiar with some resources, activities to help with teaching Stage 7 & 8 Fractions.

- Make sense of, and apply contexts to, e.g.
  $^{2}/_{3} \times {}^{4}/_{5}$ and $2\frac{1}{2} \div \frac{1}{2}$

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