Place Value Domain

This has been reduced for today. The whole thing is 24 pages long. Email me if you want a copy. (Email in footer below.)

Stage 2: Counting from One on Materials

Optional: Numbers from 10 to 19:

The key idea is that students practise bundling and connecting the bundles to symbols e.g. 17 is read as seventeen, and one ten and seven ones, and modelled as one ten and seven, and also seventeen singletons. Students need to group materials so they eventually understand that one ten and ten ones are the same.

Materials

- Sticks with rubber bands
- Beans and plastic bags
- Pipe cleaners
- Unifix, ideally with a wrapper around ten cubes - but not Multilink as small children find them hard to join together

Problems

Base problems on going from any of six ideas to any of the other five in the "Read, Say, Do Times Two" model. The ideas are:

- *Read* e.g. 13 is "thirteen"
- *Read* e.g. thirteen is "thirteen"
- *Say* in one way e.g. thirteen is "thirteen"
- *Say* in the other way e.g. thirteen is "one ten and three ones"
- *Do* in one way e.g. show 13 by counting out thirteen lollies
- *Do* in the other way e.g. show thirteen lollies as a bag of ten lollies and three loose lollies

Stage 3: Counting from One by Imaging

Essential: Numbers from 10 to 19:

See Stage 2 above

Optional: Numbers from 10 to 99

The key idea is that students practise bundling and connecting the bundles to symbols e.g. 37 is read as thirty-seven, and three tens and seven ones, and modelled as three tens and seven ones, and also thirty-seven singletons. Students need to group materials so they eventually understand that one ten and ten ones are the same.

Materials

- Pipe cleaners
- Unifix, ideally with a wrapper around ten cubes (rather than Multilink as small children find them hard to join together)

Problems

Base problems on going from any of six ideas to any of the other five in the "Read, Say, Do Times Two" model. The ideas are:

- *Read* e.g. 23 is "twenty-three"
- *Read* e.g. twenty-three is "twenty-three"
- *Say* in one way e.g. twenty-three is "twenty-three"
- *Say* in the other way e.g. twenty-three is "two tens and three ones"
- *Do* in one way e.g. show twenty-three by counting out twenty-three lollies

- *Do* in the other way e.g. show twenty-three as two bags of ten lollies and three loose lollies

Stage 4: Advanced Counting

Essential: Numbers from 10 to 99

See stage 3 above

Essential: Addition and Subtraction

Materials

- One, ten and hundred dollar play money

Problems

Problems should be solved with play money only. Mental methods are delayed until the next stage.

- Addition of two digit numbers – renaming done using play money. For example 89 + 67
- Subtraction of two digit numbers – decomposition using play money. For example 67 - 49

Stage 5: Early Part-Whole Addition and Subtraction

All activities from stage 5 onwards are essential not optional.

Numbers from 100 to 999

Repeat reading, saying, and modelling of two-digit numbers, but increase the largest numbers involved three-digit numbers.

Problems

Base problems on the "Read, Say, Do Times Two" model.

The Slavonic Abacus

Material

- The Slavonic abacus - it is a very powerful aid to help students understand place-value

Problems

- Hide the Slavonic abacus from the students, and push across, say, 67 beads – see Figure 1, page 1. Briefly show this to prevent students counting by tens. Students need to recognise there are five rows of ten and another row of ten i.e. sixty in all, and also see a row with five ones and two ones in it i.e. seven ones. So altogether there are 67 beads. Repeat with more two-digit numbers until students can recognise any two-digit number quickly.
- Proceed to imaging e.g. the teacher hides, say, 82 and asks the students what it looks like.
- Hide the Slavonic abacus from the students and push, say, 56 beads across to the right. Tell the students that you can see 56 and ask them to imagine what is on the left hand side - here it is 44. Connect 56 and 44 to 56 + 44 = 100
- Repeat frequently for pairs that add up to 100
- Given two numbers under 100 write in words stories involving pairs like bigger and smaller, largest and smallest

Mixing Words and Symbols

A mix of words and numbers is a powerful indicator as to whether students understand place-value or are merely using an algorithm that has no meaning for them. Use materials to work out the answers. The problems in this set do not require renaming.

Materials

- Preferably play money (if the students are confidently part-whole in their thinking)
- PV blocks (if the students are not confidently part-whole in their thinking)

Problems

- 50 + 30
- Thirty + forty + 20
- 90 – 10
- 60 + twenty-seven
- 420 + 300
- 230 + 400
- twenty-eight + forty
- Sixty – 20
- 24 – 10
- 200 plus 4 hundred
- 30 + 35
- 420 + sixty
- 24 + sixty
- 70 + 20
- Thirty + 83
- 3 hundred + 4 hundred
- twenty-three + fifteen
- 430 + three hundred

Mental Addition Involving "Tidy" Numbers

Problem like 67 + 6 appear to be reasonably easy but many students who struggle with part-whole thinking are overwhelmed with the complexity of such problems.

In order to simplify this an important kind of problem students need to scaffolded through using the "tidy number involved.

Materials

- Ten frames
- One, tens, and hundreds play money

Problems

- 28 + 2 = ☐
- 4 + 36 = o
- 27 + ☐ = 30
- twenty - ☐ = 17 ……

Continue doing these kinds of problems until students are fluent in doing them mentally. Mastery of this kind of problem is essential before attempting the next set of problems below.

Problems

- 28 + 8 = ☐
- 8 + 36 = ☐
- 27 + ☐ = 33
- 23 - ☐ = 17 ……

The next set of problems do not require the renaming of tens and ones. Consequently they are *not* advanced additive problems.

Standard Form for Addition

The standard algorithm for multi-digit addition in the vertical form, in which students engage in the internal talk of place-value, is a powerful aid that helps students understand place-value. For example, for 56 + 78 the self-talk would be something like this:

- Six ones and eight ones equals fourteen ones
- Swap this for one ten and four ones
- One ten plus five tens plus seven tens makes thirteen tens
- Swap this for one hundred and three tens
- The answer is one hundred and thirty-four

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  1
 56
+78
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134
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After mastering two digit addition students should go on to three digits plus three digit problems.

Stage 6: Advanced Part-Whole Addition and Subtraction

Inbetweenness and Ordering

Surprising as it may seem students who successfully find numbers between say 35 000 and 36 000 cannot solve the problem if the number are presented in word form i.e. find numbers between 35 thousand and 36 thousand. Such a mistake is highly indicative that the students are seriously lacking in their understanding of place-value, and that they are unlikely to leave school numerate. Considerable effort is needed with a mixture of symbols and words to remedy this problem.

Problems

- Find two numbers between 54 and 61
- Find two numbers between sixty-eight and ninety-seven
- Find four numbers between 18 and thirty-five
- Find three numbers between sixty-eight hundred and ninety-seven

Mental Strategies

Mental multi-digit addition and subtraction problems that have multiple methods of solution are now practiced.

Material

- Number lines - notice this is the first time number lines need to be used

Problems

- 28 + ☐ = 92 • 72 + ☐ = 91 • 27 + ☐ = 74 • 45 - ☐ = 17 ……

Continue doing these kinds of problems until students are fluent in doing them mentally.

Problems

- ☐ + 28 = 92 • ☐ + 72 = 91 • ☐ + 27 = 74 • ☐ - 23 = 38 ……

Continue doing these kinds of problems until students are fluent in doing them mentally.

Problems

- 78 + 28 = ☐ • 34 + 68 = ☐ • 88 - 39 = ☐ • 123 - 36 = ☐ ……

Continue doing these kinds of problems until students are fluent in doing them mentally.

Problems

- 100 001 - 999 989 = ☐
- 30 054 – 29 996 = ☐
- 600 023 - ☐ = 599 987
- ☐ – 89 996 = 10 004

Stage 7: Advanced Part-Whole Multiplication and Division

Multiplicative Place-Value Relationships

Problems

- A bank has only ten-dollar notes, and one dollar coins. How is $587 withdrawn?

This connects the canonical form 587 with the non-canonical form 58 tens and 7 ones.

- Model 1947 in the canonical form on play money and show why it is also read as the non-canonical form nineteen hundred and forty-seven

It is non-canonical as there is nineteen in the hundreds column – and only a maximum of 9 is permitted

- Model the non-canonical form twenty-three hundred and thirty-seven on play money and do the exchanges that show its canonical form is one thousand, three hundred and thirty-seven (i.e. 1337)

Long Division

Problems

- Solve up to five-digit numbers divided by a single digit:
  345 ÷ 5 is done by realising 345 also means 34 tens plus 5 ones

Dealing with remainders to create the fractional part of an answer is covered in the relevant fraction work.

Checking Calculations

Problems

- $67 \times 88 \neq 5016$

This is because $70 \times 90 = 6300$, and 6300 is miles away from 5016.

- 56×74 is approximately what?